Total No. of Questions : 9]

SEAT No. :

PB-3604

[Total No. of Pages : 4

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S.E. (Civil)

ENGINEERING MATHEMATICS - III

(2019 Pattern) (Semester - III) (207001)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question No. 1 is compulsory. Answer Q2 or Q3, Q4 or Q5, Q6 or Q7, Q8 or Q9.*
2) *Figures to the right indicate full marks.*
3) *Non-programmable electronic pocket calculator is allowed.*
4) *Assume Suitable data, if necessary*
5) *Neat diagrams must be drawn wherever necessary.*

Q1) Attempt the following :

i) Standard deviation of three numbers 9, 10 and 11 is **[2]**

a) $\frac{2}{3}$ b) $\frac{1}{3}$

c) $\sqrt{\frac{2}{3}}$ d) $\sqrt{2}$

ii) If $\bar{a} = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\bar{b} = \hat{i} - \hat{j} + 2\hat{k}$ then angle between $\bar{a}$ & $\bar{b}$ is _________. **[2]**

a) $\cos\left(\frac{2}{3\sqrt{6}}\right)$ b) $\cos^{-1}\left(\frac{2}{3\sqrt{6}}\right)$

c) $\cos^{-1}\left(\frac{2}{3}\right)$ d) $\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$

iii) For $\bar{F} = x^2\,\hat{i} + xy\,\hat{j}$ the value of $\int_C \bar{F}.d\bar{r}$ for curve $y^2 = x$ joining points (0, 0) and (1, 1) is - **[2]**

a) $\frac{1}{12}$ b) $\frac{7}{12}$

c) $\frac{5}{12}$ d) $\frac{2}{3}$

P.T.O.

iv) Two dimentional heat flow equation in steady state condition is **[2]**

a) $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

b) $\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x} + \frac{\partial^2 u}{\partial y^2} \right)$

c) $\frac{\partial u}{\partial t} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$

d) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

v) The vector product of two vectors is a - **[1]**

a) Vector

b) Scalar

c) Neither vector non scalar

d) none of these

vi) A card is drawn from a well shuffled a pack of 52 cards, probability of getting a club card is - **[1]**

a) $\frac{1}{4}$

b) $\frac{3}{4}$

c) $\frac{1}{3}$

d) $\frac{1}{2}$

Q2) a) If $\Sigma f = 27$, $\Sigma fx = 91$, $\Sigma fx^2 = 359$, $\Sigma fx^3 = 1567$, $\Sigma fx^4 = 7343$. Find first four moments about origin also find coefficient of skewness and kurtosis. **[5]**

b) From a record of analysis of correlation data the following results are available variance of x is 9 and lines of regression are $8x - 10y + 66 = 0$, $40x - 18y = 214$, Find (i) mean values of x & y series (ii) coefficient of correlation between x & y series (iii) standard deviation of y series. **[5]**

c) If ten percent of articles from a certain machine are defective. Waht is probability that then shall be 6 detective in a sample of 25? **[5]**

OR

Q3) a) Obtain the regression line y on x for following data. **[5]**

x	5	1	10	3	9
y	10	11	5	10	6

b) Find probability that almost 5 detective fuses will be found in a box of 200 fues if 2% of such fuses are defective. **[5]**

c) In a sample of 1000 cases the mean of a certain test is 14 and standard deviation is 2.5. Assuming that the distribution is normal find (i) How many students score between 12 and 15? (ii) How many score above 18? **[5]**

[Given :- A(0.8) = 0.2881, A(0.4) = 0.1554, A(1.6) = 0.4452)

Q4) a) For the curve $x = e^t \cos t, y = e^t \sin t, z = e^t$. Find the velocity and acceleration of the particle moving on the curve at $t = 0$. **[5]**

b) Find the directional derivative of $\phi = xy^2 + yz^3$ at $(1, -1, 1)$ along the direction normal to the surface $x^2 + y^2 + z^2 = 4$ at $(1, 2, 2)$ **[5]**

c) Show that the vector field $\vec{F} = (3x^2 y + yz)\hat{i} + (x^3 + xz)\hat{j} + xy\hat{k}$ is irrotational. Find scalar potentional ϕ such that $\vec{F} = \nabla\phi$.

OR

Q5) a) If the vector field $\vec{F} = (x+2y+az)\hat{i} + (6x - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational. Find a, b, c and determine ϕ such that $\vec{F} = \nabla\phi$ **[5]**

b) Attempt any one : **[5]**

i) $\nabla \cdot \left(\dfrac{\vec{a} \times \vec{r}}{r} \right) = 0$

ii) $\nabla^4 \cdot \left(r^2 \log r \right) = \dfrac{6}{r^2}$

c) Find directional derivative of $xy^2 + yz^2$ at $(2, -1, 1)$ along the line $2(x - 2) = (y + 1) = (z - 1)$ **[5]**

Q6) a) Use Green's lemma to evaluate the integral

$$\oint_C \left[\left(2x^2 - y^2\right) dx + \left(x^2 + y^2\right) dy \right]$$

where C is the curve bounding $y \geq 0$ and $x^2 + y^2 \leq 1$ **[5]**

b) Evaluate $\iint_S \left(\nabla \times \bar{A}\right) \cdot d\bar{S}$ where S is the surface of the cone $z = 2 - \sqrt{x^2 + y^2}$ above the xy plane and $\bar{A} = (x - z)\hat{i} + (x^3 + yz)\hat{j} - 3xy^2\hat{k}$. **[5]**

c) Evaluate the surface integral $\iint_S curl\ \bar{F} \cdot d\bar{S}$ by transforming it into a line integral, S being that part of the paraboloid $z = 1 - x^2 - y^2$ for which $z \geq 0$ and $\bar{F} = y\hat{i} + z\hat{j} + x\hat{k}$. **[5]**

OR

Q7) a) Find the value of $\int_C \bar{F} \cdot d\bar{r}$ where C is part of the spiral $\bar{r} = (a\cos\theta,\ a\sin\theta,\ a\theta)$ from $\theta = 0$, to $\theta = \dfrac{\pi}{2}$ and where $\bar{F} = r^2\hat{i}$ **[5]**

b) Obtain the equation of stream lines in case of steady motion of fluid defined by velocity $\bar{q} = \left(x^2 + y^2\right)\hat{i} + 2xy\hat{j} + \left(x + y\right)z^3\hat{k}$ **[5]**

c) Using Gauss Divergence theorem show that $\oint_S \nabla r^2 \cdot d\bar{S} = 6V_0$ where S is a smooth closed surface in the three dimensional space which contains a region of space whose numerical volume is V_0. **[5]**

Q8) a) A tightly stretched string of length l is initially in equilibrium position is set vibrating by giving to each of its points, the velocity $\left.\frac{\partial y}{\partial t}\right|_{t=0} = V_0 \sin^3\left(\frac{\pi x}{l}\right)$ find $y(x, t)$ if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ **[8]**

b) Solve $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ if, **[7]**

i) $u(0, t) = 0$

ii) $u(l, t) = 0$,

iii) $u(x, t)$ is bounded

iv) $u(x,0) = \frac{u_0 x}{l}$ for $0 \le x \le l$

OR

Q9) a) If $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ represents vibrations of string of length l, fixed at both ends, find the solution if, **[8]**

i) $y(0, t) = 0$

ii) $y(l, t) = 0$

iii) $\left.\frac{\partial y}{\partial t}\right|_{t=0} = 0$

iv) $y(x, 0) = k(lx - x^2)$ $0 \le x \le l$

b) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to conditions **[7]**

i) $u = 0$ as $y \to \infty$ $\forall_x$

ii) $u = 0$ if $x = 0$ $\forall_y$

iii) $u = 0$ if $x = l$ $\forall_y$

iv) $u = u_0 \sin\frac{\pi x}{l}$ if $y = 0$ for $0 < x < l$